Equivalent Formulas for Global Magnetic Force Calculation from Finite Element Solution

A. Demenko, W. Lyskawinski, R. Wojciechowski
Poznań University of Technology
ul. Piotrowo 3A, 60-965 Poznań, Poland
andrzej.demenko@put.poznan.pl

Abstract — The paper presents the Lorentz force and Maxwell's stress formulas that give the same result of global force calculation for FE method. Edge element method using vector potential *A* and nodal element method using scalar potential Ω are considered. The formulas have been obtained from virtual work principle that has been adopted to the FE model. The FE network must be regular to obtain the equivalent formulas. The results of force calculation using proposed methods have been compared with analytical results. The system with three cuboidal magnets has been analyzed. Moreover the forces in the system given by TEAM Workshops Problem No. 7 have been considered.

I. Introduction

The different methods have been used to describe the global magnetic force such as virtual work principle (VP), Maxwell's stresses (MS), Lorentz formula, equivalent currents (EC) [1,2,3]. For the exact solution of field equations all these methods lead to the same result and the methods are considered to be equivalent. However the commonly used FE packages do not guarantee the identity of the results, e.g. the result of MS differs from the result of EC. The paper deals with the equivalence of global force description for FE method. Edge element method (EEM) using magnetic vector potential $\boldsymbol{A}$ and nodal element method (NEM) using scalar potential Ω are considered.

II. Virtual Work Approach to the FE Solution

The force acting on the region V_D with body D is analyzed. Let us assume that we have solved FE equations with the unknown vector $\boldsymbol{\varphi}$ of edge values of $\boldsymbol{A}$ or with unknown vector $\boldsymbol{\Omega}$ of nodal values of Ω. Using these vectors we can describe the field sources (FSs) that are caused both by the conducting currents and by equivalent magnetizing currents that model ferromagnetic material and permanent magnets. For EEM the field sources represent *mmf*s $\boldsymbol{\theta}$ associated with element edges, and for NEM the sources are given by nodal flux injections $\boldsymbol{\Phi}$,

$$\boldsymbol{\theta}=\boldsymbol{C}^T\boldsymbol{R}_{\mu0}\boldsymbol{C}\boldsymbol{\varphi}\,, \quad \boldsymbol{\Phi}=\boldsymbol{K}^T\boldsymbol{\Lambda}_{\mu0}\boldsymbol{K}\boldsymbol{\Omega}\,. \qquad \text{(1a,b)}$$

Here $\boldsymbol{C}$ is the matrix that transposes vector $\boldsymbol{\varphi}$ into the vector of facet values of flux densities, $\boldsymbol{K}$ is the matrix that transforms $\boldsymbol{\Omega}$ into vector of edge values of gradΩ and $\boldsymbol{R}_{\mu0}$ or $\boldsymbol{\Lambda}_{\mu0}$ is the reluctance or permeance matrix of EEM or NEM equations that are calculated for permeability μ equal to vacuum permeability μ_0.

First, in order to describe the magnetic force acting on D the virtual principle was applied and the virtual displacement of FSs has been considered, e.g. the virtual displacement in the direction of axis x, see Fig.1. In the FE models we should consider the discrete displacement, e.g. displacement of $\pm h_x$. In the virtual displacement of FSs the FE network should be kept constant. This requirement can be satisfied for the discrete systems of regular grid in displacement direction, e.g. see Fig. 1 where height of the elements in the direction of x are identical and $h_{xi}=h_{xi+1}=h_x$. For virtual displacement in the x-direction virtual work principle gives the following formulas

$$F_x=\boldsymbol{\varphi}^T(\boldsymbol{k}_+\boldsymbol{\theta}-\boldsymbol{k}_-\boldsymbol{\theta})/(2h_x)\,,\quad F_x=\boldsymbol{\Omega}^T(\boldsymbol{k}_+\boldsymbol{\Phi}-\boldsymbol{k}_-\boldsymbol{\Phi})/(2h_x)\,, \qquad \text{(2a,b)}$$

where $\boldsymbol{k}_+$, $\boldsymbol{k}_-$ are the conversion matrices which project the displacement of *mmf*s $\boldsymbol{\theta}$ or flux injections $\boldsymbol{\Phi}$ by a distance $\pm h_x$ in the direction of axis x.

III. Force Density and Maxwell Stress Formulas

The presented above formulas (2) can be transformed into the unified Lorentz force formulas that describe the force by its volume density f using: (a) equivalent magnetizing and conducting currents for vector potential formulation and (b) equivalent nodal flux injections for scalar potential formulation. In the obtained formulas the force F_x acting on the region of the i-th element is described as follows

$$F_x=\frac{-1}{4h_x}\sum_{p=0}^{1}\sum_{q=1}^{2}(\theta_{zq,i+p}\phi_{yq,i}+\theta_{yq,i+p}\phi_{zq,i})\,, \qquad \text{(3a)}$$

$$F_x=\frac{1}{8h_x}\sum_{q=1}^{4}(u_{xq,i}\Phi_{q,i}+u_{xq,i}\Phi_{q,i+1})\,. \qquad \text{(3b)}$$

Here $\theta_{yq,i+p}$, $\theta_{zq,i+p}$ (q=1,2; p=0,1) are the components of vector $\boldsymbol{\theta}$ related to edges $e_{yq,i+p}$, $e_{zq,i+p}$; $\phi_{yq,i}$, ϕ_{zqi} are the facet values of $\boldsymbol{B}$ for facets $\boldsymbol{S}_{xq,i}$, $\boldsymbol{S}_{zqi}$; $\Phi_{q,i}$, $\Phi_{q,i+1}$(q=1,..,4) are the components of vector $\boldsymbol{\Phi}$ for nodes $Q_{q,i}$, $Q_{q,i+1}$, $u_{xq,i}$ is the edge value of $\boldsymbol{H}$ for edge $e_{xq,i}$, see Fig.1.

For regular grid the force formulas (3) can be transformed into the MS formulas that describe the mean values of stress tensor components in elements. For the i-th element in Fig. 1 we obtain

$$T_{xxi}=0.5\mu_0^{-1}(B_{x1,i}B_{x2,i}-0.5(B_{y1,i}^2+B_{y2,i}^2+B_{z1,i}^2+B_{z2,i}^2))\,, \qquad \text{(4a)}$$

$$T_{yxi}=0.25\mu_0^{-1}(B_{y1,i}+B_{y2,i})(B_{x1,i}+B_{x2,i})\,, \qquad \text{(4b)}$$

$$T_{xxi}=0{,}5\mu_0(0{,}25\sum_{q=1}^{4}H_{xq,i}^2-0{,}5\sum_{q=1}^{2}(H_{yq,i}H_{yq,i+1}+H_{zq,i}H_{zq,i+1}))\,, \qquad \text{(5a)}$$

$$T_{yxi}=\frac{\mu_0}{8}[(H_{x2,i}+H_{x4,i})(H_{y2,i}+H_{y2,i+1})+(H_{x1,i}+H_{x3,i})(H_{y1,i}+H_{y1,i+1})]. \qquad \text{(5b)}$$

Equations (4a,b) relate to the EEM and (5a,b) to the NEM, $B_{uq,i}$ (q=1,2; u=x,y,z) is the value of the u-th component of flux density $\boldsymbol{B}$ for facet $\boldsymbol{S}_{uq,i}$ and $H_{uq,i}$ is the value of the u-th component of $\boldsymbol{H}$ for edge $e_{uq,i}$ - Fig.1.

It should be noticed that in (4a) the product $B_{x1i}B_{x2i}$ represents the mean value of B_x^2 for the i-th element, $B_{xi}^2=B_{x1,i}B_{x2,i}$. However in the classical approaches mean value of B_x^2 is

$$B_{xi}^2=(B_{x1,i}+B_{x2,i})^2/4 \quad \text{or} \quad B_{xi}^2=(B_{x1,i}^2+B_{x2,i}^2)/2\,. \qquad \text{(6a,b)}$$

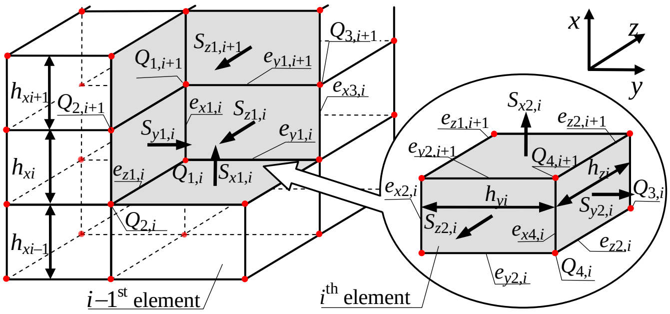

Fig.1. FE network of regular elements in the direction of axis x

The mean values of H_y^2 and H_z^2 in (5a) are represented by the products of edge values of $\boldsymbol{H}$ for edges $e_{yq,i}$, $e_{yq,i+1}$ and $e_{zq,i}$, $e_{zq,i+1}$. The description of the mean value of B_y^2 in (4a) and H_y^2, H_z^2 in (5a) is the particular feature of the presented MS formulas.

The formulas (4), (5) have been obtained from (3). Thus it is easy to prove that if the region V_D is empty and subdivided into regular elements the force calculated from (4), (5) is exactly equal to zero, i.e. (4), (5) give a faultless result (with rounding error accuracy). This is the most advantageous property of the presented methods.

IV. Examples

The presented formulas has been verified be comparing the FE solution with the analytical solution. The forces acting in the system of cuboidal permanent magnets (PMs) have been analyzed [4]. The magnets are placed in the unbounded empty space. The permeability of PMs is assumed to be equal to μ_0. The results of force calculation using FE method and formulas (2-5) have been compared with the exact analytical results. In order to obtain the analytical results a special software was prepared. The analytical method presented in [5] has been generalized and adopted for multi-magnet system. Here the results for system of 3 PMs are discussed (Fig. 2). The relative value of force F_x acting on PM I is calculated. This value is defined as follows $F_{xr}=F_x/(H_c^2\mu_0 lw)$, where H_c is the coercive force and lw is the area of active surface of PM I. Here, we present the selected results for system of $w=l=2w_a+\Delta w$, $\delta=0.3w$, $h=0.4w$. The calculations have been performed for different relative values of Δw and constant value of $2w_a+\Delta w$. First the models of regular mesh in PM region are considered. In the models $h_y=h_z=l/24$, $h_x=h/8=\delta/6$. The grid consists of about 1.4×10^6 hexahedrons. Fig. 3 demonstrates the exact values of F_{xr} and the relative error ε in F_{xr} calculation using FEM. Three methods have been applied: (a) EEM with formulas (4), (b) EEM with classical procedure using (6b), and (c) NEM with formulas (5). Is should be noticed that the results of methods (a) and (c) are independent of the location of the integration surface around D, e.g. the results for integration over the surface in the i-th and the i–1st element in Fig.1 are the same. The procedure (b) does not satisfy this property.

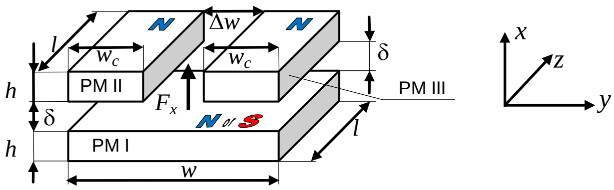

Fig.2. Considered system with 3 cuboidal permanent magnets (PM)

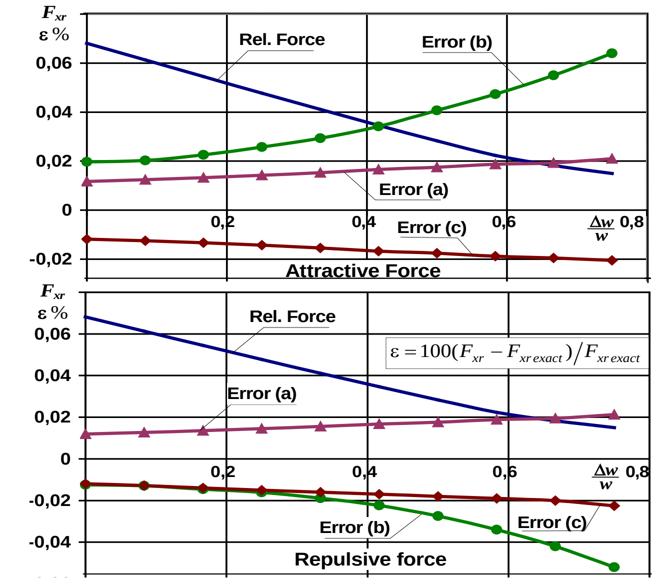

Fig.3. Relative force and error ε of force calculation for methods (a), (b), (c)

The discussed MS formulas have been used in the calculations of force between the coil and conducting plate with hole, see TEAM Problem No. 7 [6]. The A-T-T_0 formulation with EEM was applied [6]. The calculations of force have been performed for different values of distance b between the integration plane S and the upper boundary surface of plate (S lies between coil and plate). For all considered values of b formulas (4) give identical result. The maximum value of repulsive force F_r is 2.5493 N. However classical approaches with (6) give different values, e.g. from (6a) we obtain F_r=2.5357 N for b=9 mm, F_r=2.5278 N for b=21 mm, F_r=2.5122 N for b=27 mm, and from (6b) F_r=2.5228 N for b=9 mm, F_r=2.5064 N for b=21 mm, F_r=2.4750 N for b=27 mm. Thus the values of F_r differ even of 2%.

V. Conclusion

The presented methods of force calculation provide good accuracy. In the region with regular mesh the calculated value of force acting on the empty space is exactly equal to zero. The results are independent of the location of integration surface. The most important shortcoming of equivalent formulas is the requirement of homogeneity in relation to the FE grid. However even for non-regular mesh the methods give satisfactory results. The equations of EEM/NEM are similar to the equations of cells method (CM) and finite integration technique (FIT) [6]. Therefore, the presented formulas of force calculation can be easy adopted for CM and FIT.

VI. References

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